NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. Which of the following sentences are statements, which are predicates, and which are neither?

(a) 17 is an even number.

(b) n is a prime number.

(c) If n is a prime number then n is odd.

(d) Is 13 a prime number?

(e) Every even number is the sum of two odd numbers.

2. Let A, B and C be statements. Construct truth tables for the following statements. For each statement, state whether it is a tautology, a contradiction or neither.

(a) $(A \implies B) \vee (A \implies \sim B)$.

(b) $(\sim A \implies B) \vee (A \implies B)$.

(c) $(A \implies B) \wedge (\sim A \implies \sim B)$.

(d) $A \wedge \sim (A \implies B)$.

3. For any natural number n, let $A(n)$ be the statement

"If n is prime then $n+1$ is not prime."

(a) Write down the contrapositive of $A(n)$.

(b) Write down the converse of $A(n)$.

(c) Write down the negation of $A(n)$.

(d) Is $A(n)$ true for some natural number n? If so, give an example, if not give a proof.

(e) Is $A(n)$ true for every natural number n? If so, give a proof, if not give a counterexample.

(f) Is the contrapositive of $A(n)$ true for some natural number n? Is it true for every natural number n? Give brief reasons for your answer.

(g) Is the converse of $A(n)$ true for some natural number n? Is it true for every natural number n? Give brief reasons for your answer.

4. Let $f : \mathbb{N} \to \mathbb{N}$ be given by $f(x) = x^2 + 2x$.

(a) Use a **direct proof** to show that if n is even then $f(n)$ is even.

(b) Use a **proof by contraposition** to show that if $f(n)$ is even then n is even.

(c) Use a **proof by contradiction** to show that if $f(n+k)$ is odd then n is odd or k is odd.