445.255 Principles of Mathematics
Study Guide, Second Semester 2000

This is a one semester, 2 point paper, taught at the City Campus.

This paper aims to give a broad introduction to mathemetical thinking and communication, rather than technique. The main thrust is not so much finding the right answer to a problem as convincing someone else (or yourself) that the answer must be right.

By the end of this paper, you will be familiar with the basics of how to go about proving something, you will be used to seeing a new definition and deriving simple consequences of that definition, and you will have met widely used mathematical objects like groups and equivalence relations. You will also have learned more about some familiar structures: the natural numbers and the real numbers. All this will prepare you well for Stage 3 papers in Pure Mathematics. It should also prepare you to tackle new real-life problems instead of limiting yourself to problems someone else has already solved for you!

Because the nature of the course is quite different from what you have seen before, the teaching and assessment methods will also be new. An important part of what you are learning is the ability to *communicate* mathematics. Some time during lectures will be given over to group discussions, where you will be expected to decide and to convince your fellow students of how to prove the propositions on the blackboard. You will also be expected to participate in "Collaborative Tutorials": see below for details.

Syllabus.

Logic (0.5 weeks): Statements, variables, theorems, truth tables and other basic ideas of formal logic.
Set Theory (0.5 weeks): Sets, basic set operations, De Morgan's laws.
Relations and Orderings (1 week): Relations, (partial) orders, partitions and equivalence relations.
Functions (1 week): The formal definition of a function, composition, inverse.
Induction (1 week): The axiom of induction and complete induction.
Elementary Number Theory (1.5 week): The natural numbers and integers, divisibility, factorization, the Euclidean algorithm, congruence mod n.
Algebra of polynomials (0.5 weeks): Division of polynomials, Euclidean algorithm for polynomials.
Groups (2 weeks): Binary operations. Groups defined. Examples. Subgroups, group homomorphisms, quotient groups and cosets.
Real Numbers (1 week): Axioms for the real numbers, total ordering, least upper bounds, Archimedian property, convergence of sequences.
Introduction to Real Analysis (3 weeks): Formal definitions of continuity and differentiability. Uniform continuity, Taylor's theorem.

Prerequisites. One of 445.130, 445.152, 445.109, 445.208 or equivalent.

Calculators. Calculators will be permitted in the Test and the Examination.

Course Material. The text is *Chapter Zero Fundamental Notions of Abstract Mathematics* by C. Schumacher (Addison-Wesley). The course will follow this book closely, so we strongly recommend that you purchase your own copy. It will be available on short loan in the Science Library.

Assessment. This will be based on assignments (30%), the semester test (10%) and the final exam (60%), OR on assignments (10%), the semester test (10%) and the final exam (80%), whichever is higher.

Note that the test and coursework count at least 20% towards your final mark.

Work will be marked for its clarity and precision as well as its content. For example, a proof poorly expressed with symbols undefined might be failed even if the "idea" of the proof is correct. Conversely, if a proof is well set up it might gain a pass mark even if the method is completely wrong. We encourage students to work together on the assignments, but remember that what you hand in must be your own work!

The time and date of the **Semester Test** will be announced in class. No makeup test will be given.

The ten **assignments** are to be handed in by 4pm on the following Wednesdays: 26 July, 2 August, 9 August, 16 August, 23 August, 13 September, 20 September, 27 September, 4 October, 11 October.

The Student Resource Centre will not accept late assignments under any circumstances. Assignments placed in the wrong box will not be marked, so be careful where you put your work. The overall assignment mark will be based on these assignments and on marks in the collaborative tutorials (see below). Each assignment will be worth 30 marks and each collaborative tutorial will be worth 10 marks, to give a total out of 360 marks.

Lectures and Tutorials. There are two streams, at 9am each weekday and at 3pm each weekday. The Tuesday–Friday sessions will be lectures.

Tutorial sessions will be held every Monday (except the first day of the semester). There will be five **'regular' tutorials** with an emphasis on review and working through problems and six **'collaborative' tutorials** where students work in groups of three on a problem to be handed in at the end of the session for marking.

The collaborative tutorials are run as follows: a tutorial assignment is completed during the tutorial working in groups of 3 (although we may accept groups 2 or 4 people if necessary). You don't have to be in the same group each time.

There will be a few short questions designed to be able to be answered in about 40-45 minutes. 10 minutes before the end you will be required to put your answers onto provided sheets which are handed in. A brief rundown of the answers is then given on an overhead.

Summary of Important Dates. R is a 'regular' tutorial, C is a 'collaborative' tutorial, A is an assignment due date. The date of the test is to be advised.

Week	Dates	Mon	Tues	Wed	Thurs	Fri
1	17–21 July					
2	24–28 July	C1		A1		
3	31 July–4 August	R1		A2		
4	7–11 August	C2		A3		
5	14–18 August	R2		A4		
6	21–25 August	C3		A5		
	28 August–8 September	B	R	E	A	K
7	11–15 September	R3		A6		
8	18–22 September	C4		A7		
9	25–29 September	R4		A8		
10	2–6 October	C5		A9		
11	9–13 October	R5		A10		
12	16–20 October	C6				

Office hours. You are encouraged to approach your lecturers with any questions or suggestions you have about the paper. Specific office hours will be announced in class.

Teaching staff.

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Paper Homepage. The WWW homepage for this paper is

`http://www.math.auckland.ac.nz/~class255`.

From here you will be able to download copies of the assignments and other handouts, and obtain information about your coursework marks.