1. (a) The contrapositive of $A(n)$ is "If n is even then n is not prime".

(b) The converse of $A(n)$ is "If n is odd then n is prime".

(c) The negation of $A(n)$ is "n is prime but n is not odd".

(d) $A(n)$ is true for some $n \in \mathbb{N}$, for example $A(5)$ is true.

(e) $A(n)$ is not true for every $n \in \mathbb{N}$, for a counterexample $A(2)$ is false.

(f) By (d) and (e) we know that the contrapositive of $A(n)$ is true for some n but not for every n.

(g) The converse of $A(n)$ is true for some n but not for every n. For example the converse of $A(5)$ is true, but the converse of $A(9)$ is false.

2. We must show that $\sim$ is reflexive, symmetric and transitive.

Reflexive: Let $x \in S$. Then, since ρ is reflexive, $x \,\rho\, x$ (and $x \,\rho\, x$), so $x \sim x$.

Symmetric: Let $x, y \in S$ with $x \sim y$. Then $x \,\rho\, y$ and $y \,\rho\, x$, so $y \,\rho\, x$ and $x \,\rho\, y$, so $y \sim x$.

Transitive: Let $x, y, z \in S$ with $x \sim y$ and $y \sim z$. Then $x \,\rho\, y$ and $y \,\rho\, x$, and $y \,\rho\, z$ and $z \,\rho\, y$. Since $x \,\rho\, y$ and $y \,\rho\, z$ and ρ is transitive, $x \,\rho\, z$. Similarly, since $z \,\rho\, y \,\rho\, x$ we have $z \,\rho\, x$. So $x \,\rho\, z$ and $z \,\rho\, x$, so $x \sim z$.

3. (a) $n = 2 : \displaystyle\sum_{i=2}^{2} \frac{1}{i(i+1)} = \frac{1}{2(3)} = \frac{2-1}{2(2+1)}$. True for $n = 2$.

Assume $k \geq 2$ and that $\displaystyle\sum_{i=2}^{k} \frac{1}{i(i+1)} = \frac{k-1}{2(k+1)}$. Then

$$\begin{aligned}\sum_{i=2}^{k+1} \frac{1}{i(i+1)} &= \frac{k-1}{2(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{(k-1)(k+2)+2}{2(k+1)(k+2)} = \frac{k^2+k-2+2}{2(k+1)(k+2)} \\ &= \frac{(k+1)(k)}{2(k+1)(k+2)} = \frac{k}{2(k+2)}.\end{aligned}$$

By Principle of Mathematical Induction, true for all $n \geq 2$.

(b) Let $\emptyset \neq B \subseteq \mathbb{Z}$ and assume B is bounded below by c. We may assume $c < b$ for all $b \in B$. Let $B' = \{b - c : b \in B\}$. (that is, we subtract c from everything to get a subset of $\mathbb{N}$). Then $\emptyset \neq B' \subseteq \mathbb{N}$, so B' has a smallest element, say b'. Then $b = b' + c \in B$, and b is smallest in B since b' is smallest in B'.

4. (a) $a|b \Rightarrow b = na$ for some $n \in \mathbb{Z}$.
$a|c \Rightarrow c = ma$ for some $m \in \mathbb{Z}$. Suppose $d = \gcd(b, c)$, say $d = bx + cy$ for some $x, y \in \mathbb{Z}$. Then $d = nax + may = a(nx + my)$ so $a|d$.

(b) $a \equiv b \bmod n \Rightarrow n|b - a \Rightarrow b - a = nr$.
$c \equiv d \bmod n \Rightarrow n|c - d \Rightarrow c - d = ns$.
$ac - bd = ac - ad + ad - bd = a(c-d) + (a-b)d = ans - nrd = n(as - rd)$. So $n|ac - bd$ and $ac \equiv bd \bmod n$.

(c) Solving $\overline{21} \cdot_{50} \overline{x} = \overline{1}$ is the same as solving $21x \equiv 1 \text{mod} 50$, or finding x, y such that $21x + 50y = 1$.

50	1	0
21	0	1
8	1	−2
5	−2	5
3	3	−7
2	−5	12
1	8	−19

So $1 = (21)(-19) + (50)(8)$. The solution is $\overline{x} = \overline{-19} = \overline{31}$.