THE UNIVERSITY OF AUCKLAND **445.255 SC**

TERMS TEST

MATHEMATICS

Principles of Mathematics

(Time allowed: 90 MINUTES)

1. Let n be a natural number, and let $A(n)$ be the statement

"If n is prime then n is odd".

(a) Write down the contrapositive of $A(n)$.

(b) Write down the converse of $A(n)$.

(c) Write down the negation of $A(n)$.

(d) Is $A(n)$ true for some $n \in \mathbb{N}$? If so, give an example, if not give a proof.

(e) Is $A(n)$ true for every $n \in \mathbb{N}$? If so, give a proof, if not give a counterexample.

(f) Is the contrapositive of $A(n)$ true for some $n \in \mathbb{N}$? Is it true for all $n \in \mathbb{N}$? Give brief reasons for your answer.

(g) Is the converse of $A(n)$ true for some $n \in \mathbb{N}$? Is it true for all $n \in \mathbb{N}$? Give brief reasons for your answer.

2. Let S be a set and let ρ be a binary relation on S. Suppose that ρ is both reflexive and transitive. Define a new relation $\sim$ on S by declaring that

$$x \sim y \text{ if and only if } x \,\rho\, y \text{ and } y \,\rho\, x.$$

Show that $\sim$ is an equivalence relation on S.

3. (a) Prove that for all integers $n \geq 2$,

$$\sum_{i=2}^{n} \frac{1}{i(i+1)} = \frac{n-1}{2(n+1)}.$$

(b) Assume that $\mathbb{N}$ is well-ordered, that is, every non-empty subset of $\mathbb{N}$ has a least element. Prove that every non-empty subset of $\mathbb{Z}$ that has a lower bound has a least element.

4. (a) Recall that the greatest common divisor of positive integers b, c is the smallest positive integer of the form $bx + cy, \quad x, y \in \mathbb{Z}$. Suppose that a, b, c are positive integers and that $a|b$ and $a|c$. Prove that $a|\gcd(b, c)$.

(b) Prove that if $a \equiv b \mod n$ and $c \equiv d \mod n$, then $ac \equiv bd \mod n$.

(c) Solve the equation $\overline{21} \cdot_{50} \overline{x} = \overline{1}$ for $\overline{x} \in \mathbb{Z}_{50}$.