1. Intuitively the limit ought to be 5/3. Now $\left|\dfrac{5n}{3n+1}-\dfrac{5}{3}\right| = \dfrac{5}{3(3n+1)}$. So given $\epsilon > 0$ we need to have $\dfrac{5}{9n+3} < \epsilon$, i.e. $n > \dfrac{5-3\epsilon}{9\epsilon}$. Take $N = \left[\dfrac{5-3\epsilon}{9\epsilon}\right] + 1$. Then for $n > N$ we have $\dfrac{5}{9n+3} < \epsilon$ and so $\left|\dfrac{5n}{3n+1}-\dfrac{5}{3}\right| < \epsilon$ as required.

2. (a) Since $\{x_n\}$ is bounded there is a number $K > 0$ such that $|x_n| < K$ for all n, i.e. $-K \leq x_n \leq K$. So K is an upper bound for all the x_n. It follows that $s_n < K$ and certainly $s_n > -K$ because $s_n \geq x_n$. So $\{s_n\}$ is bounded.

(b) If $x_n \to l$ as $n \to \infty$ then given $\epsilon > 0$ there is an integer N such that $|x_n - l| < \epsilon$ for $n > N$ i.e. $l - \epsilon < x_n < l + \epsilon$ for $n > N$. So we must have $l - \epsilon < s_n < l + \epsilon$ for $n > N$ and hence $s_n \to l$ as $n \to \infty$.

3. Firstly $x_1 < 2$ and if $x_{n-1} < 2$ then $x_n < \sqrt{2.2} = 2$. So $\{x_n\}$ is bounded above by 2. Also $\dfrac{x_n}{x_{n-1}} = \sqrt{\dfrac{2}{x_{n-1}}} > 1$. So $\{x_n\}$ is increasing and bounded above and hence converges. Suppose $x_n \to l$ as $n \to \infty$. Then $l = \sqrt{2l}$ from which it follows that $l = 0$ or 2. But $x_n \geq 1$ for all n. So $l = 0$ is impossible and hence $x_n \to 2$ as $n \to \infty$.

4. Suppose $x_n \to l$ as $n \to \infty$. Then given $\epsilon > 0$ there is an integer N such that $|x_n - l| < \epsilon/2$ and $|x_m - l| < \epsilon/2$ for $n, m > N$. But then $|x_m - x_n| = |x_m - l + l - x_n| \leq |x_m - l| + |x_n - l| < \epsilon/2 + \epsilon/2 < \epsilon$. So x_n is a Cauchy sequence.

5. Let the subsequence be $\{x_{n_k}\}$. Then given $\epsilon > 0$ there is an integer N_1 such that $|x_{n_k} - l| < \epsilon/2$ for $n > N_1$. Also since $\{x_n\}$ is a Cauchy sequence there is a N_2 such that $|x_m - x_n| < \epsilon/2$ for $m, n > N_2$. Since $n_k > n$ for all k we have in particular that $|x_{n_k} - x_n| < \epsilon/2$. So if $n > N = \max\{N_1, N_2\}$ we have $|x_n - l| \leq |x_n - x_{n_k}| + |x_{n_k} - l| < \epsilon/2 + \epsilon/2 < \epsilon$ and hence $x_n \to l$ as $n \to \infty$.