1. To translate the statements and predicates into symbols, we will use $P(n)$ to denote "n is prime", $O(n)$ to denote "n is odd", $E(n)$ to denote "n is even" and $A(x, y, z)$ to denote $x + y = z$.

(a) "17 is a prime number" is a statement, which we denote by $P(17)$. [The statement is true, but that was not part of the question.]

(b) "If n is a prime number then n is odd" is a predicate, which we denote by $P(n) \implies O(n)$. [It is not a theorem: $n = 2$ is a counterexample.]

(c) "Is 13 a prime number?" is neither a statement nor a predicate.

(d) "Every even number is the sum of two odd numbers" is a statement, which we denote by

$$(\forall x)(E(x) \implies (\exists y)(\exists z)(O(y) \wedge O(z) \wedge A(y, z, x))).$$

[There are other possible answers: for example, we might define a predicate $B(n)$ to be "n is the sum of two odd numbers", and the statement becomes $(\forall n)(E(n) \implies B(n))$.]

2. (a) We have the following truth table:

A	B	$\sim B$	$A \implies \sim B$	$A \implies B$	$(A \implies \sim B) \vee (A \implies B)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	T

Since the last column contains only "T"s, the statement is a tautology.

(b) We have the following truth table:

A	B	$A \implies B$	$\sim A$	$\sim A \implies B$	$(A \implies B) \vee (\sim A \implies B)$
T	T	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

Since the last column contains only "T"s, the statement is a tautology.

(c) We have the following truth table:

A	B	$A \implies B$	$\sim A$	$\sim B$	$\sim A \implies \sim B$	$(A \implies B) \vee (\sim A \implies \sim B)$
T	T	T	F	F	T	T
T	F	F	F	T	T	T
F	T	T	T	F	F	T
F	F	T	T	T	T	T

Since the last column contains only "T"s, the statement is a tautology.

(d) We have the following truth table:

A	B	$B \implies A$	$\sim(B \implies A)$	$A \wedge \sim(B \implies A)$
T	T	T	F	F
T	F	T	F	F
F	T	F	T	F
F	F	T	F	F

Since the last column contains only "F"s, the statement is a contradiction.

3. (a) The contrapositive of $A(n)$ is "If $n^2 + n$ is odd then n is even".

(b) The converse of $A(n)$ is "If $n^2 + n$ is even then n is odd".

(c) The negation of $A(n)$ is "n is odd and $n^2 + n$ is odd".

(d) Yes, $A(n)$ is true for some $n \in \mathbb{N}$: for example, $A(1)$ is true.

(e) Yes, $A(n)$ is true for every $n \in \mathbb{N}$.

Proof: Let $n \in \mathbb{N}$. Suppose n is odd. Then $n = 2k + 1$ for some $k \in \mathbb{Z}$, so

$$\begin{aligned} n^2 + n &= (2k+1)^2 + (2k+1) \\ &= 4k^2 + 4k + 1 + 2k + 1 \\ &= 4k^2 + 6k + 2 \\ &= 2(2k^2 + 3k + 1), \end{aligned}$$

and $2k^2 + 3k + 1 \in \mathbb{Z}$, so $n^2 + n$ is even.

(f) Since the contrapositive of $A(n)$ is equivalent to $A(n)$, from (d) and (e) we know that the contrapositive of $A(n)$ is true for some $n \in \mathbb{N}$ and that it is true for every $n \in \mathbb{N}$.

(g) The converse of $A(n)$ is true for some $n \in \mathbb{N}$: for example the converse of $A(1)$ is true. However, it is not true for all n: for example, the converse of $A(2)$ is false.

4. There are several possible answers. The most obvious is either "For every even integer n, $(n+1)^2$ is odd" or "For every integer n, if n is even then $(n+1)^2$ is odd". [As usual, we might leave the universal quantification implicit, i.e. we might write this as "If n is even then $(n+1)^2$ is odd".]