NB: Please deposit your solutions in the appropriate box by **4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a Mathematics Department cover sheet: these are available from the Resource Centre.

1. If $f(x) = x^2(\sin\frac{1}{x} + \cos 5x)$ for $x \neq 0$ and $f(0) = 0$ prove *from the definition* that f is continuous at 0.

2. (a) $a_0, a_1, \ldots, a_{n-1}$ are given constants. Prove that

$$\left|\frac{a_0}{x^n} + \frac{a_1}{x^{n-1}} + \cdots + \frac{a_{n-1}}{x}\right| \leq \frac{1}{2}$$

if $|x|$ is large enough.

(b) Deduce for (a) that the equation $x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$ has a root if n is odd.

Hint: Write the LHS as $x^n\left(1 + \cdots + \frac{a_0}{x^n}\right)$ and use part (a) and Bolzano's theorem.

3. Let $f(x) = \begin{cases} x^2 & x \text{ irrational} \\ 0 & x \text{ rational.} \end{cases}$

(a) Prove from the definition that f is is differentiable at $x = 0$ and $f'(0) = 0$.

(b) Prove that f is not even continuous at $x = \sqrt{2}$. (In fact it is not continuous at any other point.)

4. If f is such that f''' (the 3rd derivative) exists in $[a, b]$ and

$$f(a) = f'(a) = f(b) = f'(b) = 0$$

show that there is a point p in (a, b) for which $f'''(p) = 0$.