NB: Please deposit your solutions in the appropriate box by **4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a Mathematics Department cover sheet: these are available from the Resource Centre.

1. Prove from first principles (i.e. directly from the definition) that the sequence $\left\{ \dfrac{5n}{3n+1} \right\}$ converges.

2. Let $\{x_n\}$ be a bounded sequence of real numbers and let

$$s_n = \operatorname{lub}\{x_n, x_{n+1}, \ldots\} \qquad n = 1, 2, \ldots$$

   (a) Show that the sequence $\{s_n\}$ is bounded.

   (b) If $x_n \to l$ as $n \to \infty$ show that $s_n \to l$ as $n \to \infty$.

3. $\{x_n\}$ is a sequence in $\mathbb{R}$ for which $x_1 = 1$ and $x_n = \sqrt{2x_{n-1}}$ for $n > 1$. Show that $\{x_n\}$ converges and find its limit.

4. Show that every convergent sequence is a Cauchy sequence.

5. If $\{x_n\}$ is a Cauchy sequence which has a subsequence that converges to a limit l show that $\{x_n\}$ also converges to l.