

**NB:** Please deposit your solutions in the appropriate box by **4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a Mathematics Department cover sheet: these are available from the Resource Centre.

1. If  $a < b + \epsilon$  for every  $\epsilon > 0$  prove that  $a \leq b$ .
2. (a) Prove that  $|a + b| \leq |a| + |b|$ .  
(b) Find the values of  $x$  in  $\mathbb{R}$  for which  $\left| \frac{x + 3}{3x - 2} \right| < 5$ .
3. Prove that the set  $S = \left\{ \frac{n - 1}{2n} : n \in \mathbb{N} \right\}$  is bounded above. What is the least upper bound? Does  $S$  have a maximum?
4. If  $A, B$  are nonempty subsets of  $\mathbb{R}$  show that  $\text{lub}(A + B) = \text{lub}A + \text{lub}B$ . If also  $A \subset B$  and  $B$  is bounded above show that  $\text{lub}A \leq \text{lub}B$ .
5. Let  $A$  be a nonempty subset of  $\mathbb{R}$ . Suppose that  $x \in A$  is an upper bound for  $A$ . Prove that  $x = \text{lub}A$ .
6. If  $x, y \in \mathbb{R}$  prove that there is an irrational number  $s$  such that  $x < s < y$ .  
(Hint: Think how to get a specific small positive irrational number.)