NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a Mathematics Department cover sheet: these are available from outside the Resource Centre.

1. Prove that for all positive integers a, b, c

$$\text{lcm}(ca, cb) = c\text{lcm}(a, b).$$

[Hint: Using the Fundamental Theorem of Arithmetic, represent each of a, b, c as products of the same prime factors with non-negative exponents.]

2. Suppose that

$$n = \prod_{i=1}^{r} p_i^{e_i}$$

is the standard factorization of the positive integer n. That is, $p_1, \cdots, p_r$ are distinct primes and each $e_i > 0$.

(a) How many positive divisors of n are there?

(b) Show that n may be factored into the product of two relatively prime factors in 2^{r-1} different ways.

3. (a) Find the remainder when 65^{67} is divided by 11. [Hint: $65 \equiv -1$ mod11.]

(b) Find the remainder when 13^{189} is divided by 5.

4. (a) Find all solutions in $\mathbb{Z}_n$ to the equation $\bar{6} = \bar{4} \cdot_n \bar{x}$ for

(i) $n = 8$

(ii) $n = 10$

(iii) $n = 15$

(b) Show that $\overline{143}$ has an inverse in $\mathbb{Z}_{368}$ and find it.

5. Let $a(x), b(x)$ be the polynomials in $\mathbb{Q}[x]$ defined by

$$a(x) = x^4 + 2x^3 - x^2 + 4x - 6, \quad b(x) = x^5 + 2x^2 - 4x + 4.$$

Find the greatest commmon divisor $d(x)$ of $a(x), b(x)$ and find polynomials $u(x), v(x)$ such that

$$d(x) = a(x)u(x) + b(x)v(x).$$