

**NB:** Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. Prove by induction that for all  $n \in \mathbb{N}$ ,

$$\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

2. Prove by induction that for all  $n \in \mathbb{N}$ ,

$$\sum_{i=1}^n \frac{i}{2^i} = 2 - \frac{n+2}{2^n}.$$

3. Prove that if  $A, B$  are sets with  $n, m$  elements respectively, then there are exactly  $m^n$  distinct possible functions  $f : A \rightarrow B$ . [Hint: Use induction on  $n$ .]
4. Suppose that in a pile of 2 or more coins it is known that one is heavier than the rest but all the others are the same weight. Prove that four weighings with a balance are sufficient to discover which coin is heavier. [Hint: Set one coin aside and consider the possibilities for the rest.]