1. Define the relation $\sqsubseteq$ on the Cartesian plane $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ by

$$(x, y) \sqsubseteq (u, v) \text{ if and only if } x \leq u \text{ and } y \leq v.$$

(a) Which of the following are true?

(i) $(1, 1) \sqsubseteq (2, 3)$.

(ii) $(1, -1) \sqsubseteq (2, -2)$.

(iii) $(1, 2) \sqsubseteq (2, 1)$.

(iv) $(2, 1) \sqsubseteq (1, 2)$.

(b) Prove that $\sqsubseteq$ is a partial order on $\mathbb{R}^2$.

(c) Is $\sqsubseteq$ a total order on $\mathbb{R}^2$? Give a brief reason for your answer.

2. Let $f : A \to B$ be a function. Define a relation $\sim_f$ on A by declaring that, for $x, y \in A$,

$$x \sim_f y \qquad \text{if and only if} \qquad f(x) = f(y).$$

Show that $\sim_f$ is an equivalence relation on A.

3. Define a relation $\sim$ on the Cartesian plane $\mathbb{R}^2$ by declaring that, for $(u, v), (x, y) \in \mathbb{R}^2$,

$$(u, v) \sim (x, y) \qquad \text{if and only if} \qquad 3u - v = 3x - y.$$

(a) Show that $\sim$ is an equivalence relation on $\mathbb{R}^2$.

(b) Give a geometric description of the equivalence class $T_{(0,0)}$ of the point $(0, 0)$.

(c) Give a geometric description of the equivalence class $T_{(u,v)}$ of the point (u, v).

(d) Give a geometric description of the set $\mathcal{R}_\sim$ of equivalence classes under $\sim$.

4. Let $f : A \to B$ and $g : B \to C$ be functions. Suppose that $g \circ f$ is onto and g is one-to-one. Show that f is onto.