

1. Let G be the set of integers with a binary operation $a \cdot b = 5a - 4b$. Establish whether or not (G, \cdot) is a group.
2. Let $G = (U(18), \cdot_{18})$. Construct a Cayley table for G . Verify that G forms a group.
3. Let $(G, *)$ be a group. Assume that $(a * b)^2 = a^2 * b^2$ for all $a, b \in G$. Prove that G is abelian.
4. Let $S = M_{2 \times 2}(\mathbb{R})$ be the set of all 2×2 matrices with entries from the real numbers. Describe whether or not S forms a group under the operation of matrix multiplication.
5. Let S be the set of all real numbers except -1 . Define an operation $*$ on S by $a * b = a + b + ab$.
 - (a) Prove that $*$ is a binary operation on S .
 - (b) Show that $(S, *)$ is a group.