NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. Which of the following sentences are statements, which are predicates, and which are neither? Translate all the statements and predicates into symbols.

(a) 17 is a prime number.

(b) If n is a prime number then n is odd.

(c) Is 13 a prime number?

(d) Every even number is the sum of two odd numbers.

2. Let A, B and C be statements. Construct truth tables for the following statements. For each statement, state whether it is a tautology, a contradiction or neither.

(a) $(A \implies \sim B) \vee (A \implies B)$.

(b) $(A \implies B) \vee (\sim A \implies B)$.

(c) $(A \implies B) \vee (\sim A \implies \sim B)$.

(d) $A \wedge \sim(B \implies A)$.

3. For any integer n, let $A(n)$ be the statement

"If n is odd then $n^2 + n$ is even".

(a) Write down the contrapositive of $A(n)$.

(b) Write down the converse of $A(n)$.

(c) Write down the negation of $A(n)$.

(d) Is $A(n)$ true for some $n \in \mathbb{N}$? If so, give an example, if not give a proof.

(e) Is $A(n)$ true for every $n \in \mathbb{N}$? If so, give a proof, if not give a counterexample.

(f) Is the contrapositive of $A(n)$ true for some $n \in \mathbb{N}$? Is it true for all $n \in \mathbb{N}$? Give brief reasons for your answer.

(g) Is the converse of $A(n)$ true for some $n \in \mathbb{N}$? Is it true for all $n \in \mathbb{N}$? Give brief reasons for your answer.

4. Consider the following proof:

Let n be an even integer. Then $n = 2m$ for some integer m, so $n + 1 = 2m + 1$, so $(n+1)^2 = (2m+1)^2 = 4m^2 + 4m + 1 = 2(2m^2 + 2m) + 1$. Thus $(n+1)^2$ is odd.

What have we proved?