THE UNIVERSITY OF AUCKLAND **445.255 FC**

TERMS TEST

MATHEMATICS

Principles of Mathematics

(Time allowed: 90 MINUTES)

1. Let n be an integer, and let $A(n)$ be the statement

"If n is greater than 5 then n^2 is greater than 35".

(a) Write down the contrapositive of $A(n)$.

(b) Write down the converse of $A(n)$.

(c) Write down the negation of $A(n)$.

(d) Is $A(n)$ ever true? If so, give an example, if not give a proof.

(e) Is $A(n)$ always true? If so, give a proof, if not give a counterexample.

(f) Is the contrapositive of $A(n)$ ever true? Is it always true? Give brief reasons for your answer.

(g) Is the converse of $A(n)$ ever true? Is it always true? Give brief reasons for your answer.

2. Define the relation $\sim$ on the Cartesian plane $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ by

$$(u, v) \sim (x, y) \text{ if and only if } u + y = v + x.$$

(a) Give a geometric description of the set $T_{(0,0)}$ of relatives of the point $(0, 0)$. [A well-labelled diagram will be sufficient.]

(b) Give a geometric description of the set $T_{(u,v)}$ of relatives of the point (u, v).

(c) Give a geometric description of the collection $\mathcal{R}_{\sim} = \{T_{(u,v)} : u, v \in \mathbb{R}\}$ of subsets of $\mathbb{R}^2$ associated with $\sim$.

(d) Show that $\sim$ is an equivalence relation on $\mathbb{R}^2$.

3. Let A and B be sets, and let $f : A \to B$ be a function.

(a) Suppose that there is a function $g : A \to B$ such that $(g \circ f)(x) = x$ for all $x \in A$. Show that f is one-to-one.

(b) Suppose that there is a function $h : A \to B$ such that $(f \circ h)(y) = y$ for all $y \in B$. Show that f is onto.

(c) Using (a) and (b), show that f has an inverse if and only if f is a one-to-one correspondence.

4. (a) Let $(X, \leq)$ and $(Y, \sqsubseteq)$ be partially ordered sets, and let $f : X \to Y$ be an order isomorphism. Show that for any $x \in X$, x is a maximal element of X if and only if $f(x)$ is a maximal element of Y.

(b) Let $X = (0, 1)$ and $Y = [0, 1]$ be subsets of $\mathbb{R}$, with the usual ordering $\leq$. Show that X and Y are not order isomorphic.

(c) Show that the subsets $Z = (0, 1] \cup (2, 3)$ and $W = (0, 2)$ of $\mathbb{R}$ are order isomorphic. [Hint: you will need to write down a function $f : Z \to W$ and show that it is an isomorphism. Your definition of $f(x)$ will probably have two cases, depending on whether x is in $(0, 1]$ or $(2, 3)$.]