NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet. These are available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Show that $8^n - 5^n$ is a multiple of 3 for every $n \in \mathbb{N}$. [Hint: $8^{n+1} - 5^{n+1} = 8 \cdot 8^n - 5 \cdot 8^n + 5 \cdot 8^n - 5 \cdot 5^n$.]

2. We say that a function $f : \mathbb{R} \to \mathbb{R}$ is a *wibble* function if, for every $x \in \mathbb{R}$, $f(2x) = 2f(x)$. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are functions, $f + g : \mathbb{R} \to \mathbb{R}$ is defined by $(f + g)(x) = f(x) + g(x)$ for all $x \in \mathbb{R}$.

(a) Show that if $f : \mathbb{R} \to \mathbb{R}$ is a wibble function then $f(2^n x) = 2^n f(x)$ for every $x \in \mathbb{R}$ and every $n \in \mathbb{N}$.

(b) Show that if f and g are both wibble functions then $f + g$ is a wibble function.

(c) What is the converse of the result in part (b)?

(d) Show that the converse of the result in part (b) is false.

3. Show that if $a, b \in \mathbb{N}$ and $d = \gcd(a, b)$ then there exist integers x and y with $d = ax + by$. [Hint: use complete induction, and the fact that if $a = qb + r$ then $\gcd(a, b) = \gcd(b, r)$. Your proof should begin with something like "For $b \in \mathbb{N}$, let P_b be the statement that for every $a \in \mathbb{N}$ there exist integers x and y with $d = ax + by$"]

4. Let $a, b \in \mathbb{N}$. Show that a and b are relatively prime if and only if there exist $x, y \in \mathbb{Z}$ such that $ax + by = 1$.

5. Use Euclid's Algorithm to show that 53 and 25 are coprime. Using your working, or otherwise, find integers x and y such that $53x + 25y = 1$.