NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet. These are available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Let $(x_n)_{n=1}^{\infty}$ and $(y_n)_{n=1}^{\infty}$ be sequences of real numbers. Define the sequence $(z_n)_{n=1}^{\infty}$ by $z_n = x_n + y_n$.

(a) Show that if (x_n) and (y_n) are both increasing sequences then (z_n) is an increasing sequence. What is the converse of this statement? Does the converse hold? If it holds, give a proof, otherwise give a counterexample.

(b) Give an example of monotonic sequences (x_n) and (y_n) such that (z_n) is not monotonic.

2. If A and B are sets, we define the *symmetric difference* of A and B, $A \triangle B$, by

$$A \triangle B = (A \setminus B) \cup (B \setminus A) = \{\, x : x \in A \text{ or } x \in B \text{ but not both} \,\}.$$

(a) Show that for any sets A, B and C, $A \triangle (B \triangle C) = (A \triangle B) \triangle C$.

(b) Let S be a set. Show that $(\mathcal{P}(S), \triangle)$ is an abelian group. [Hint: for any A we have $A \triangle \varnothing = A$ and $A \triangle A = \varnothing$.]

3. Let $(G, *)$ be a group, and let $H \subseteq G$. Show that H is a subgroup of G if and only if

- $e \in H$ (where e is the identity element of G);
- for any $x, y \in H$, $x * y \in H$; and
- for any $x \in H$, $x^{-1} \in H$.

4. Let $(G, *)$ be a group, and let $H \subseteq G$. Show that H is a subgroup of G if and only if $H \neq \varnothing$ and, for every $x, y \in H$, $x * y^{-1} \in H$.

5. Let $(G, *)$ and $(H, \diamond)$ be groups. A *group homomorphism* from G to H is a function $f : G \to H$ such that for all $x, y \in G$,

$$f(x * y) = f(x) \diamond f(y).$$

If $f : G \to H$ is a group homomorphism then the *kernel* of f, $\ker(f)$, is defined by

$$\ker(f) = \{\, x \in G : f(x) = e_H \,\},$$

where e_H is the identity element of H. Show that $\ker(f)$ is a subgroup of G.