NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet. These are available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. (a) Consider the partial ordering $\leq$ on the set $X = \{1,2,3,4,5,6\}$ defined by $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(2,1),$
$(2,3),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(6,4),(5,4)\}$.

Draw the lattice diagram.

List the sets of maximal, minimal, greatest and least elements of A. (Some of these sets may be empty, or have several elements.) Give reasons.

(b) Now let A be a set, and consider the power set of A, $\mathcal{P}(A)$, with the partial ordering $\subseteq$.

Prove that $A \in \mathcal{P}(A)$ is a maximal element of $\mathcal{P}(A)$. Does $\mathcal{P}(A)$ have a greatest element?

Give one example of a nonempty set A where $\subseteq$ is a total ordering, and another where $\subseteq$ is not a total ordering.

2. Define a relation on $\mathbb{Z}$ by $a \sim b \iff (a-b)/3 \in \mathbb{Z}$.

Describe the set of relatives of $1 \in \mathbb{Z}$ (ie the set T_1).

Write down $\mathcal{R}_\sim \subseteq \mathcal{P}(\mathbb{Z})$,and justify your answer.

Prove that $\sim$ is an equivalence relation by showing that $\mathcal{R}_\sim$ is a partition of $\mathbb{Z}$.

Describe the equivalence classes.

Hint: Recall (from a previous assignment) that every integer x can be written in exactly one of the following ways (with $k \in \mathbb{Z}$): $3k$, $3k+1$ or $3k+2$.

3. Let $f : A \to B$ and $g : B \to C$ be functions.

Prove that if both f and g are one-to-one, then $g \circ f$ is one-to-one.

Let X and Y be subsets of A. Prove that $f(X) \backslash f(Y) \subseteq f(X \backslash Y)$.

Give an example where $f(X) \backslash f(Y) \neq f(X \backslash Y)$. (Be careful to define f properly and to show it is an example.)

4. Let $X = [0,1) \subseteq \mathbb{R}$ and $Y = [1,3) \subseteq \mathbb{R}$. Let $f : X \to Y$ be defined by $f(x) = 2x+1$.

Prove that f is a 1-1 correspondence.

The usual ordering $\leq$ on $\mathbb{R}$ gives orderings on X and Y. (In other words, we can compare elements of X, say, because they are real numbers.) Prove that f is an order isomorphism.

5. Let A be a set and consider the binary operation on $\mathcal{P}(A)$ given by set difference: $a * b = a \backslash b$ for $a, b \in \mathcal{P}(A)$.

Is this operation associative?

Is this operation commutative?

Give proofs or counterexamples.