NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet. These are available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Let $A = \{0, 1\}$ and $B = \{1, 2, 3\}$. Write down, $\mathcal{P}(A)$ and $\mathcal{P}(B)$, the power sets of A and B.

Does $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$?

Does $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$?

Prove that, for all sets X and Y, $\mathcal{P}(X) \subset \mathcal{P}(Y) \implies X \subset Y$.

Hint: Prove the following 2 statements, then show (using the definition of a proper subset) that they imply the result.

$\mathcal{P}(X) \subseteq \mathcal{P}(Y) \implies X \subseteq Y$ and

$\mathcal{P}(X) \neq \mathcal{P}(Y) \implies X \neq Y$.

2. (a) Prove for any sets A, B and C that if $C \subseteq B$ then $A \times C \subseteq A \times B$.

Draw a picture of $\mathbb{R} \times \mathbb{N}$ as a subset of $\mathbb{R} \times \mathbb{R}$. (Use labels!)

(b) Let $\rho = \{(a, b) \in \mathbb{R} \times \mathbb{R} : (a - b)/2 \in \mathbb{N}\}$. Draw ρ as a subset of $\mathbb{R} \times \mathbb{R}$. Describe the relation defined by ρ.

(Eg: "Integer a is related to integer b if and only if $a + b$ is a prime number." would be a good answer for *another* relation.)

3. Define relations (a) to (e) as follows.

(a) $\sim$ is a relation on $\mathbb{Z}$, with $a \sim b$ iff $a - b \in \mathbb{N}$.

(b) Let A be a set. $\sim$ is a relation on $\mathcal{P}(A)$, with $a \sim b$ iff $a \subset b$.

(c) $\sim$ is a relation on $\mathbb{Q}$, with $a \sim b$ iff $ab \in \mathbb{Z}$.

(d) $\sim$ is a relation on $\mathbb{R}$, with $a \sim b$ iff $a + b = 0$.

(e) $\sim$ is a relation on $\mathbb{Z}$, with $a \sim b$ iff $a - b < 1$.

Fill in a copy of the following table. (In other words, indicate if each relation has each of the properties listed.)

Relation	Reflexive	Symmetric	Antisymmetric	Transitive
a		$\star$		
b			$\star$	
c				
d				$\star$
e	$\star$			

For each box that has a star, justify your answer. (Either prove that the relation has the property, or give a counterexample.)

4. Let $A = \{1, 2, 3, 4, 5\}$, and let $S = \{(5, 2), (4, 1), (4, 3), (1, 2)\} \subseteq A \times A$.

Construct a set $T \subseteq A \times A$ with the following 3 properties.

1. $S \subseteq T$.
2. T is a partial ordering on the set A.
3. T has a minimal number of elements.

Hint: Add the ordered pairs to S that are required to ensure that the new set defines a partial ordering.

Draw the lattice diagram for T.

Now add ordered pairs to the set T to obtain another relation U that is a total ordering on A. (This can be done in more than 1 way.)

Draw the lattice diagram for U.