NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet. These are available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. For each of the sentences, answer the following questions.

Is it a statement? A predicate?

List the free and bound variables (if any).

Express each symbolically in terms of the statements, predicates, sets and relations given.

Decide if it is a theorem,tautology or contradiction (or none of these).

If the statement is a theorem, prove it. If it is false, give a counterexample.

(a) "A or B but not both" in terms of 'A' and 'B'.

(b) "Either exactly one of the statements A, B and C is true, or they all are" in terms of 'A', 'B' and 'C'.

(c) "Everybody loves somebody sometime" in terms of the set of all people 'P', the set of all times 'T', and the predicate R(x,y,t) = 'person x loves person y at time t'.

(d) "Nobody loves everybody all the time" using the same notation as part c.

(e) "Given integers a and b satisfying $a < b$, there is a rational number c such that $a < c$ and $c < b$" in terms of the sets 'Q' and 'Z' (rational numbers and integers) and the predicate R(x,y) = 'the rational number x is less than the rational number y'.

2. Construct truth tables for the following compound statements. Are either of them tautologies? Rewrite them using only the symbols A , B , C ,˜ ,∨ and ∧. If possible, simplify them. (The truth tables may help here.)

$(A \Longrightarrow B) \wedge \tilde{}(A \Longleftrightarrow C)$

$\tilde{}(\tilde{}(A \vee \tilde{}B) \wedge \tilde{}(\tilde{}A \vee B))$

3. (a) Prove "for all $n \in Z$, 3 divides $n \Longleftrightarrow$ 3 divides n^3 " using a proof by cases.

You can assume that every integer can be written in exactly 1 of the following forms (with $k \in Z$): $3k$, $3k+1$, $3k+2$.

Note: $a \in Z$ divides $b \in Z$ if there exists $c \in Z$ such that $ac = b$.

(b) Now use the first part and a proof by contradiction to prove that there is no $d \in Q$ satisfying $d^3 = 3$.

You may use the fact that every rational number can be expressed in the form a/b where $a, b \in Z$ have no common factor. (Meaning no integers except 1 and -1 divide both a and b.)

4. Prove the following theorems from set-theory.

(a) $B \backslash (B \backslash A) = A \cap B$.

(b) $(A \cup B^c)^c = A^c \cap B$.

(c) Every set except the empty set has a proper subset.